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# A NOTE ON SYMBOLIC LOGIC AND THE LAW

ROBERT S. SUMMERS \*

In recent years, several writers have sought to show how symbolic logic might be useful to lawyers.<sup>1</sup> Foremost among these is Mr. Layman Allen, who presently offers a course at the Yale Law School on applications of symbolic logic to law.<sup>2</sup>

The ensuing discussion consists of a brief explanation and criticism of some of Mr. Allen's work. The conclusion of this discussion is that symbolic logic, as used by Mr. Allen, is of inconsequential value to most lawyers. Despite this conclusion, and perhaps some bias, the writer believes that he has, nevertheless, presented enough of the flavor of Mr. Allen's work to enable readers to make up their own minds without an excessive expenditure of time.

Traditionally, logicians have focused their study on the forms of valid reasoning.<sup>3</sup> To carry on this study, they have evolved the organon of symbolic logic, which is used primarily to exhibit the internal structure of propositions and to expose formal relationships between propositions.<sup>4</sup> Mr. Allen has borrowed a part of this new apparatus and put it to an entirely new use unrelated to the study of formal reasoning. He has sought primarily to show how some of the conventions of symbolic logic can be utilized in discovering ambiguity in legal documents and authorities, and has claimed that:<sup>5</sup>

By using . . . [symbolic logic] . . . a draftsman can more exactly express his intended meaning, so that those who must interpret and apply the instrument need not speculate as much about probable intention. At the same time, the draftsman will be alerted against the inadvertent inclusion of ambiguity, which may lead to unnecessary litigation. Furthermore, in the interpretation of instruments drafted in the traditional

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<sup>1</sup> Allen, *Symbolic Logic: A Razor-Edged Tool for Drafting and Interpreting Legal Documents*, 66 YALE L.J. 833 (1957); Allen & Orechkoff, *Toward a More Systematic Drafting and Interpreting of the Internal Revenue Code: Expenses, Losses, and Bad Debts*, 25 U.CHI.L.REV. 1 (1957); Tammelo, *Sketch for a Symbolic Juristic Logic*, 8 J.LEGAL ED. 277 (1956); Montrose, *Language of, and a Notation for, the Doctrine of Precedent*, 2 U.W.AUSTL.ANN.L.REV. 301 (1953); Oppenheim, *Outline of a Logical Analysis of Law*, 11 PHIL. OF SCI. 142 (1944).

<sup>2</sup> The description of this course in the 1959-60 Yale Law School catalogue is "Symbolic Logic and Legal Communication."

<sup>3</sup> In recent years, professional logicians have tended to concentrate their efforts on highly abstract forms of reasoning rather than on practical reasoning of the type that occurs in legal discourse and everyday argument. For a thorough indictment of this development, see STEPHEN E. TOULMIN, *THE USES OF ARGUMENT* (1958).

<sup>4</sup> For an excellent description of the elements of symbolic logic, see PETER FREDERICK STRAWSON, *INTRODUCTION TO LOGICAL THEORY* esp. 1-66 (1952).

<sup>5</sup> Allen, *Symbolic Logic: A Razor-Edged Tool for Drafting and Interpreting Legal Documents*, 66 YALE L.J. 833, 855 (1957).

manner, systematic pulverization can be used to discover the wide variety of possible interpretations that are logically available.

The "symbolic logic" Mr. Allen adapts to his use consists of a symbolic notation and a four-step procedure that he calls "systematic pulverization." The notation is used to symbolize the logical syntax of propositions that are then, by "systematic pulverization," restated as compound propositions of the form "if P, then Q."

Mr. Allen explains his symbolic notation as follows:<sup>6</sup>

The system of logic upon which systematic-pulverization is based deals with the logical connections expressed in ordinary English by such words as "and," "or," "if . . . then . . ." and "not." The technical names of the logical connections expressed by these words and the symbols that represent them in systematically-pulverized form are conjunction (&), inclusive disjunction (&OR), exclusive disjunction (OR), implication (—————), coimplication (—————) and negation (NOT).

Mr. Allen describes "systematic pulverization" as follows:<sup>7</sup>

The process of transforming an ordinary statement in a natural language into systematically-pulverized form may be conveniently classified into four steps:

- A. Pulverize the statement into its constituent elements.
- B. Rearrange the constituent elements into approximately the form of an implication (or coimplication).
- C. Determine the appropriate schematic form by ascertaining the appropriate logical connection intended by the words used in the statement.
- D. Write the statement in systematically-pulverized form.

To enable readers to grasp the full flavor of systematic-pulverization, it is necessary to quote one of Mr. Allen's examples at some length:<sup>8</sup>

An illustration of one of the most commonly overlooked ambiguities—whether the connection between two elements of a statement is intended to be implication or coimplication—is found in section 65 of the Uniform Sales Act:

"SEC. 65 WHEN SELLER MAY RESCIND CONTRACT OR SALE

Where the goods have not been delivered to the buyer, and the buyer has repudiated the contract to sell or sale, or has manifested

<sup>6</sup> Allen & Orechkoff, *Toward a More Systematic Drafting and Interpreting of the Internal Revenue Code: Expenses, Losses and Bad Debts*, 25 U.CHI.L.REV. 1, 2 (1957).

For the sake of clarity, it is helpful to elaborate the meaning of several of the logical connectives as follows:

Inclusive disjunction: either A or B or both

Exclusive disjunction: either A or B but not both

Implication: if A, then B

Coimplication: if A, and only if A, then B

<sup>7</sup> *Id.* at 8.

<sup>8</sup> Allen, *Symbolic Logic: A Razor-Edged Tool for Drafting and Interpreting Legal Documents*, 66 YALE L.J. 833, 855-57 (1957).

his inability to perform his obligations thereunder, or has committed a material breach thereof, the seller may totally rescind the contract or the sale by giving notice of his election so to do to the buyer."

The essential idea in systematically-pulverizing a proposition like section 65 is to separate the statement into its constituent elements and then to determine the appropriate logical relationships between them. One convenient breakdown of section 65 is the following:

- A = Where the goods have NOT been delivered to the buyer
- B = the buyer has repudiated the contract to sell OR the sale
- C = the buyer has manifested his inability to perform his obligations thereunder
- D = the buyer has committed a material breach thereof
- E = the seller gives notice of his election to rescind to the buyer
- F = THE SELLER MAY TOTALLY RESCIND THE CONTRACT OR SALE.

The context indicates that the ambiguous "or" connecting B, C and D is an inclusive disjunction so that on its face the statute says:

9.1	1.	A				
	&2.	B	&OR	C	&OR	D
	&3.	E				
	4.	F				

Section 65 clearly declares that if the other antecedents are satisfied then:

IF (E) the seller notifies,  
THEN (F) the seller may rescind.

But does "expressio unius est exclusio alterius" apply to notification?<sup>22</sup> If the other antecedents are satisfied, must the seller notify the buyer of his intention before he can rescind, or are there other pathways open for the seller to gain the right to rescind in addition to the one explicitly expressed in section 65? If the expression of a seller's right to rescind by way of notification is intended to exclude all other possibilities, then section 65 would be interpreted as follows:

9.2	1.	A				
	&2.	B	&OR	C	&OR	D
	3.	1.	E			
		2.	F			

If the draftsman had been using systematic pulverization the question of whether 9.1 or 9.2 was intended would have been brought to his attention; he would have been reminded to indicate his choice between them, if he desired to do so. On its face section 65 does not indicate clearly which of these two interpretations was intended. For that matter, there are six other possible ways of interpreting the logical relationships between F and the other elements:

9.3	1.	A						
	&2.	E						
	3.		1.	B	&OR	C	&OR	D
			2.					
			F					
9.4	1.	B	&OR	C	&OR			
	&2.	E						
	3.		1.	A				
			2.					
			F					
9.5	1.	A						
	2.	1.	B	&OR	C	&OR	D	
		&2.	E					
			3.					
			F					
9.6	1.	B	&OR	C	&OR			
	2.		1.	A				
		&2.	E					
			3.					
			F					
9.7	1.	E						
	2.	1.	A					
		&2.	B		&OR	C	&OR	D
			3.					
			F					
9.8	1.	A						
	&2.	B	&OR	C	&OR			
	&3.	E						
	4.		F					

For a given statement the number of possible implication-coimplication interpretations of the statement can be mathematically determined. Where the number of antecedents in the statement = N, the number of possible interpretations =  $2^N$ . In this case  $N = 3$ , so the number of possible interpretations =  $2^3 = 8$ .

Thus, for what appears to be a relatively simple and straightforward statutory passage, there are often a wide variety of possible interpretations. In section 65 there are eight different combinations of implication and coimplication for a court to choose among. It is suggested that in many—but certainly not all—such cases the consensus of the legislature

would be embodied in just one of the possible interpretations, and that ought to be specified clearly, rather than expressed in the usual broad and ambiguous form. This example illustrates how systematic pulverization, by the questions it raises, can be used as a tool to lead the legislature to express more clearly just what it does intend—at least in those cases where it wishes to express a clear intention. It also illustrates the usefulness of systematic pulverization for the advocate, who is provided with a comprehensive and systematic reminder of all the possible interpretations he might argue for his client. (Citations omitted.)

From the foregoing illustration and commentary, it can be seen that a grasp of systematic pulverization would be of some utility to some lawyers when working on highly complex problems. Mr. Allen, however, has grossly overstated the value of his “razor-edged tool”:<sup>9</sup>

A large amount of the litigation based on written instruments—whether statute, contract, will, conveyance or regulation—can be traced to the draftsman’s failure to convey his meaning clearly. Frequently, of course, certain items may purposely be left ambiguous, but often the question in issue is due to an inadvertent ambiguity that could have been avoided had the draftsman clearly expressed what he intended to say. In this Article it is suggested that a new approach to drafting, using certain elementary notions of symbolic logic, can go a long way towards eliminating such inadvertent ambiguity.

Every lawyer knows that the overwhelming majority of issues concerning the meaning of terms in legal documents and authorities arise because of the vagueness or “open texture” of key concepts and not because of syntactical ambiguity. For example, disputes concerning the meaning of terms such as “sale” and “material breach” in section sixty-five of the Uniform Sales Act arise far more frequently than disputes concerning the proper interpretation of “ors” and “ands” appearing in the Act. As systematic pulverization has no application to the problems of conceptual vagueness, it cannot be used to limit what is unquestionably the most prolific source of legal disputes.

Nor can systematic pulverization uncover conceptual ambiguity.<sup>10</sup> The utility of Mr. Allen’s apparatus is confined to the discovery of ambiguity that is syntactical in nature—*i. e.*, that which arises because of the way in which words in a sentence are arranged, and this type of ambiguity is a relatively minor source of genuine legal disputes.

It should also be emphasized that the symbolic apparatus cannot, by its nature, enable a draftsman to determine whether an instance of syntactical

<sup>9</sup> *Id.* at 833.

<sup>10</sup> “. . . legal concepts are both vague and [what is different] ambiguous. . . .” H. L. A. Hart, *Analytical Jurisprudence in Mid-Twentieth Century: A Reply to Professor Bodenheimer*, 105 U.P.A.L.REV. 953, 956 (1957).

The distinction between ambiguity and vagueness is often glossed over. In the present instance, the distinction is obviously significant.

ambiguity may be legally significant. Mr. Allen unfortunately overlooks this point in the following discussion of one of his examples:<sup>11</sup>

. . . in *Chichester Diocesan Fund v. Simpson*, in which the House of Lords was called upon to construe a residuary bequest to testator's executors in trust

"for such charitable institution or institutions or other charitable or benevolent object or objects . . . as [his] . . . executors . . . may in their . . . absolute discretion select. . . ."

The important ambiguity was the word "or" used to connect the words "charitable" and "benevolent." It is a cardinal rule of English common law that a man can not delegate his testamentary power. Lord Simonds explained that there is only one exception to this rule:

"A testator may validly leave it to his executors to determine what charitable objects shall benefit, so long as charitable and no other objects may benefit."

Since the "or" was interpreted by the majority of the Lords to indicate disjunction, the executors under the will would have been empowered to distribute to objects that were benevolent but *not* charitable, and the will was thus held invalid. . . .

This was clearly a will that would have been saved if the draftsman had been using systematic pulverization. When he came to the troublesome "or" between "charitable" and "benevolent," he would have been faced with a specific choice in systematically pulverizing. He would have been forced to make a decision to represent that "or" by one of the following five symbols:

- 1- "&" indicating conjunction,
- 2- "====" indicating coimplication,
- 3- "&OR" indicating inclusive disjunction,
- 4- "OR" indicating exclusive disjunction,
- 5- "or" indicating that the draftsman wished to be ambiguous.

There is little doubt that if the draftsman had been faced with this choice, a valid will would have been written by his specifying one of the first two choices. (Citations omitted.)

Mr. Allen neglects the fact that the draftsman's error in the *Chichester* case might just as well have occurred because of an unawareness of the legal significance of the ambiguity rather than because of failure to perceive it.

It is highly questionable whether Mr. Allen's apparatus is needed to enable lawyers to see significant syntactical ambiguity. Most lawyers working on ordinary problems would probably discover in the usual course of analysis all significant ambiguities. It is difficult to resist the conclusion that Mr. Allen has devised an elaborate sledgehammer for cracking (systematically pulverizing) peanuts.<sup>12</sup>

<sup>11</sup> Allen, *Symbolic Logic: A Razor-Edged Tool for Drafting and Interpreting Legal Documents*, 66 *YALE L.J.* 833, 858 (1957).

<sup>12</sup> One of the remarks of Professor Romane L. Clark on the work of Ilmar Tammelo in the field of symbolic logic and law is appropriate here:

"His machine-tooled symbolic logic seems strangely inappropriate for cracking the common-sense chestnuts he presents us." Clark, *On Mr. Tammelo's Conception of Juristic Logic*, 8 *J.LEGAL ED.* 491, 496 (1956).

Moreover, emphasis on the role of logical words such as "or," "and," etc., might encourage adoption of that interpretation of a document or legal authority which is most consistent with the technical use of the logical words occurring in the document. Professor O. C. Jensen has pointed out:<sup>13</sup>

That the logical structure which is unambiguously indicated by the logical words in a statement should be taken as the one intended by the author of the statement is not a rule of logic. On the contrary, it is logical to give the statement the other logical structure if there is factual evidence that this is the one the author of the statement intended.

Finally, to the extent that Allen's sledgehammer is useful as insurance against lack of comprehensive analysis the question arises whether the protection afforded by this insurance is worth the premium paid. Many lawyers are likely to conclude that they do not have the time or the capacity to master and apply a technical tool of such restricted utility.

To summarize: Mr. Layman Allen's apparatus, which embodies some of the conventions of modern symbolic logic, is probably of little value to most lawyers. Although by its use syntactical ambiguity in legal documents and authorities can be exposed, this type of ambiguity is a relatively minor source of genuine legal disputes. Moreover, most lawyers can probably discover such significant ambiguity without using the apparatus; and those who cannot do this would probably also be unable to use the apparatus efficiently.

<sup>13</sup> O. C. JENSEN, *THE NATURE OF LEGAL ARGUMENT* 27 (1957).