Model of Criminal Process Game Theory and Law

Robert L. Birmingham
A MODEL OF CRIMINAL PROCESS:
GAME THEORY AND LAW

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Early ordnance was prized for both destructive and aesthetic properties. During the sixteenth century, Italian craftsmen, although aware of the attendant sacrifice of ballistic efficiency, complemented the engraving with which they adorned the field pieces they manufactured by decorating the shot itself.1 Much discussion of criminal law is analogous to the Italian cannonball; analytic embellishment adds richness in detail but often conceals crucial structural interrelationships. In this article I attempt to use the techniques of game theory to isolate minimal attributes of problems familiar in criminal law. In the first section I construct a model indicating the impact of the criminal law on the actions of prospective malefactors and, by extension, on the welfare of society as a whole; in subsequent sections I explore the implications of this model for public policy. My hope is to achieve clarity through abstractness.2

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1 C. Cipolla, GUNS AND SAILS IN THE EARLY PHASE OF EUROPEAN EXPANSION 1400-1700, at 43 (1965).

2 Introductory discussion of any field perhaps unfamiliar to the reader is hampered by the propensity of those working in it to develop a specialized vocabulary that prevents accurate definition of any single term without appeal to other terms that are equally unknown; successful explication thus necessarily tempers obscurity with imprecision. Shubik, whose stature as a scholar sanctions expository oversimplification, has provided probably the best preliminary description of the work on which I rely:

Game theory is a method for the study of decision making in situations of conflict. It deals with human processes in which the individual decision-unit is not in complete control of other decision units entering into the environment. It is addressed to problems involving conflict, cooperation, or both, at many levels. The decision-unit may be an individual, a group, a formal or an informal organization, or a society....

. . . . .

The essence of a "game" in this context is that it involves decision makers with different goals or objectives whose fates are intertwined. The individuals are in a situation in which there may be many possible outcomes with different values to them. Although they may have some control which will influence the outcome, they do not have complete control....

. . . . .

. . . The individual must consider how to achieve as much as is possible, taking into account that there are others whose goals differ from his own and whose actions have an effect on all. . . . He must adjust his plans not only to
Abstraction can, of course, be pursued to the point of disutility. Although I feel my efforts stop well short of this point, I have based my analysis on several simplifying assumptions that necessarily limit the applicability of my conclusions. Utility is considered cardinally measurable, so that numbers expressing values may legitimately indicate not only their rank but also the magnitude of the differences among them. Attention is restricted to a community composed of a sanctioning authority, termed the state, and a large number of individuals with identical tastes and beliefs, whose levels of happiness may be summed to obtain total community welfare. A single choice confronts each individual: he can do or refrain from doing a defined act condemned as criminal by the state. State response is similarly dichotomous: the sanctioning authority may impose or fail to impose a single precise penalty on the individual. Gains and losses caused by commission or noncommission of the proscribed act and punishment or nonpunishment by the state are treated as independent. While the behavior of the individual is a function of the disparity in attractiveness of his alternative opportunities, irrationality and limited powers of discrimination are assumed to render his conduct probabilistic rather than deterministic.

I

COMPONENTS OF PLEASURE

Pairs of alternative actions may be represented symbolically:

\[ a_1 = \text{noncommission of the offense by the individual} \]
\[ b_1 = \text{commission of the offense by the individual} \]

his own desires and abilities but also to the desires and abilities of others.

The outcome of the game will depend on the strategies employed by every player . . . and possibly on events beyond the control of any player . . .

Shubik, Game Theory and the Study of Social Behavior: An Introductory Exposition, in GAME THEORY AND RELATED APPROACHES TO SOCIAL BEHAVIOR 3, 8-9, 13 (M. Shubik ed. 1964).

See generally R. LUCE & H. RAFFA, GAMES AND DECISIONS: INTRODUCTION AND CRITICAL SURVEY (1957); G. OWEN, GAME THEORY (1968).


Homans, for example, attributes this assertion to Claude Lévi-Strauss: "I had looked for a society reduced to its simplest expression. That of the Nambikwara was so far reduced that I found only men there." G. HOMANS, THE NATURE OF SOCIAL SCIENCE, at xi (1967). Russell's translation unimaginatively replaces the penultimate word of the quotation with "human beings." C. LÉVI-STRAUSS, TRISTES TROPICALES 310 (J. Russell transl. 1961).
\[ a_2 = \text{nonpunishment of the individual by the state} \]
\[ b_2 = \text{punishment of the individual by the state} \]

Successive decision by the individual and the state offers four possible outcomes, each yielding the individual a stated level of utility. Let \( u \) stand for the utility to him of licit behavior unaccompanied by punishment. If, in addition, his gain in utility from committing the offense without regard to the probability of punishment and his loss in utility from punishment without regard to his conduct are indicated by \( v \) and \( w \) respectively, one may write:

\[
\begin{align*}
\text{a}_1\text{a}_2 &= \text{noncommission & nonpunishment} \rightarrow u \\
\text{b}_1\text{a}_2 &= \text{commission & nonpunishment} \rightarrow u + v \\
\text{a}_1\text{b}_2 &= \text{noncommission & punishment} \rightarrow u - w \\
\text{b}_1\text{b}_2 &= \text{commission & punishment} \rightarrow u + v - w
\end{align*}
\]

These outcomes and the values associated with them can be expressed most conveniently in what is termed normal form. Here the individual selects a row and the state a column; their choices jointly determine an outcome:

\[
\begin{array}{ccc|ccc}
& a_2 & b_2 \\
\hline
a_1 & u & u - w \\
b_1 & u + v & u + v - w
\end{array}
\]

The decision of the state to punish or not to punish any individual will normally be in part a function of whether he has committed the proscribed act; that it is not completely so is often attributable to the inability to distinguish adequately culpability from innocence. Since it is certain that the state will either punish or not punish, the probabilities of these actions must sum to unity but may differ depending upon the prior choice of the individual. Setting \( y \) equal to the likelihood of punishment given obedience to the law by the individual and \( z \) equal to the increase in this likelihood resulting from his criminal behavior, one obtains:

\[
\begin{align*}
1 - y &= \text{probability of } a_2 \text{ given } a_1 \\
1 - y - z &= \text{probability of } a_2 \text{ given } b_1 \\
y &= \text{probability of } b_2 \text{ given } a_1 \\
y + z &= \text{probability of } b_2 \text{ given } b_1
\end{align*}
\]

Combination of the probabilities of punishment and nonpunishment with the utilities of various outcomes allows computation of the expected value to the individual of doing or not doing the proscribed act. The expected value of a game that pays one dollar if a flipped
coin lands heads up and nothing if it lands tails up is fifty cents: one multiplies the gain from each outcome by its likelihood and adds the products. \( E_{a_1} \) and \( E_{b_1} \), the expected values of choices \( a_1 \) and \( b_1 \), may be similarly derived:

\[
E_{a_1} = u(1 - y) + (u - w)y
\]

\[
E_{b_1} = (u + v)(1 - y - z) + (u + v - w)(y + z)
\]

Let:

\[
1 - x = \text{probability of selection of } a_1
\]

\[
x = \text{probability of selection of } b_1
\]

The assumed identity of individual tastes and beliefs permits interpretation of \( x \) as a personal propensity or as the proportion of the population of the community behaving unlawfully. Multiplication of the expected values of equations (1) and (2) by their probabilities of realization and summation yields:

\[
E = (1 - x)E_{a_1} + xE_{b_1} = u + vx - wy - wxz
\]

Again because of the identity of tastes and beliefs, \( E \) is not only the expected value of the complete game to each individual but also an index of community welfare. Assuming benevolent government, the goal of one player, the state, reduces to maximization of the worth of the game to the other player, the individual.

II

Morality and Law

The distinction between social control through moral suasion and through rules of criminal law assumes its most Manichaean form in Chinese legal thought. Confucian philosophy admonishes the ruler to govern by exemplary personal moral excellence; the legalist tradition urges that offensive conduct must be discouraged by threat of punishment. The posited dichotomy, present but less striking in Western theory, is premised on parallels between alternative policies evident in the model.

Let the sanctioning authority be perfectly efficient, so that all those and only those who have committed the proscribed act are punished. Then:

\[\text{D. Bodde \\& C. Morris, Law in Imperial China: Exemplified by 190 Ch'ing Dynasty Cases 3-51 (1967).}\]
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\[ y = 0 \]
\[ z = 1 \]

Since the probabilities of outcomes \( a_1b_2 \) and \( b_1a_2 \) fall to zero, the game of section I reduces to choice by the individual between outcomes \( a_1a_2 \) and \( b_1b_2 \), with values to him of \( u \) and \( u + v - w \) respectively. By previous assumption one may write \( x \), the probability that an individual will behave illegally, as a function of \( v - w \), the disparity between the values of these outcomes. Hence criminality may be reduced by lowering \( v \) or raising \( w \); changes in these two variables which are equal in magnitude but opposite in sign will have identical preventive impact.

Herodotus, perhaps the least reliable of ethnographers, reported:

Another tribe further to the east is known as the Padaei, and the people live on raw meat. As to the general way of life of these folk, it is said that when a man falls sick, his closest companions kill him, because, as they put it, their meat would be spoilt if he were allowed to waste away with disease. The invalid, in these circumstances, protests that there is nothing the matter with him—but to no purpose. His friends refuse to accept his protestations, and kill and eat him just the same. Should the sufferer be a woman, it makes no odds: her women friends deal with her precisely as the men do. If anyone is lucky enough to live to an advanced age, he is offered in sacrifice and devoured—this, however, rarely happens, because most of them will have had some disease or other before they get old, and will consequently have been killed by their friends.\(^5\)

Our distaste for such practices appears almost independent of fear of criminal punishment. Socialization seems largely a consequence of early training sensitizing the individual to feelings of shame or guilt; primarily through contacts within the family, he learns to associate transgression of community norms with discomfort resulting either from the disapprobation of others or from self-blame. The model indicates a gain from guiding conduct by instilling pride in proper behavior rather than by promoting remorse concerning personal inadequacies: \( v \), inversely related to the success of the socialization process, should where possible be diminished by increasing \( u \) while holding \( u + v \) constant. The state can exercise partial control over \( v \) through its educational policies; mere designation of an act as an offense, moreover, may alter its value to the indoctrinated.\(^6\)

While protest against official failure to maintain a belief that

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“every Crime is a sinne”7 must remain essentially unfocused, specific criticism of levels of punishment normally accompanies public alarm concerning the extent of illegal behavior. Inability to enforce Prohibition in the United States inspired radical proposals:

One woman suggested that liquor law violators should be hung by the tongue beneath an airplane and carried over the United States. Another suggested that the government should distribute poison liquor through the bootleggers; she admitted that several hundred thousand Americans would die, but she thought that this cost was worth the proper enforcement of the dry law. Others wanted to deport all aliens, exclude wets from all churches, force bootleggers to go to church every Sunday, forbid drinkers to marry, torture or whip or brand or sterilize or tattoo drinkers, place offenders in bottle-shaped cages in public squares, make them swallow two ounces of castor oil, and even execute the consumers of alcohol and their posterity to the fourth generation.8

As the model demonstrates, the threat of punishment can deter the potential wrongdoer; moreover, the likelihood of such deterrence increases with the severity of the associated penalty. Punishment nevertheless need not be maximized. Although \( x \) falls as \( w \) increases, the level of \( w \) is itself a component of community welfare.

The currently topical distinction between application of additional repressive force and efforts to remedy causes of discontent is reflected in the possibility of achieving a desired disparity between \( u \) and \( u + v - w \) while holding either \( u \) or \( u + v - w \) constant. Two centuries ago Adam Smith noted that “\([n]o body will be so mad as to expose himself upon the highway, when he can make better bread in an honest and industrious manner.\)”9 He thus recognized that if gain from licit behavior is substantial, proscribed actions alternative to such behavior may prove counterproductive even without regard to the prospect of punishment; in terms of the variables of the model, a higher \( u \) may be associated with a lower \( v \). In Britain during the Industrial Revolution pauperism was consciously discouraged by reducing the amenities of the workhouse below those attainable through private employment. Since the standard of living of the free laborer was already abysmally low, imprisonment at times represented the most attractive opportunity of the unskilled; an incarcerated offender might receive twice as much food as those confined under the poor laws, while the labor required of him

7 T. Hobbes, Leviathan 151 (1651).
9 A. Smith, Lectures on Justice, Police, Revenue and Arms 156 (E. Cannan ed. 1896).
was occasionally only half as great. Here the customary excess of \( u \) over \( u - w \) was more than dissipated by the rigors of the competitive system.

While the prospect of imprisonment today seldom motivates illicit conduct, poverty remains an important factor:

In a recent year, the crime rate in Washington for the month of August jumped 18 percent over the preceding month. A veteran police officer explained the increase to David L. Bazelon, Chief Judge, U.S. Court of Appeals for the District of Columbia. "It's quite simple. . . . You see, August was a very wet month. . . . These people wait on the street corner each morning around 6:00 or 6:30 for a truck to pick them up and take them to a construction site. If it's raining, that truck doesn't come, and the men are going to be idle that day. If the bad weather keeps up for three days . . . we know we are going to have trouble on our hands—and sure enough, there invariably follows a rash of purse-snatchings, housebreakings and the like. . . . These people have to eat like the rest of us, you know."

Increasing penalties when prosperity without redistribution increases temptation towards crime produces questionable results; more adequate provision for the disadvantaged would make the criminal alternative less enticing.

III

Probabilities of Punishment

The assumption of the previous section that the state can unerringly discriminate between innocent and guilty individuals is obviously unrealistic. Packer has contrasted a Crime Control Model, characterized by the overriding importance of repression of deviant behavior, with a Due Process Model, where emphasis is placed on avoiding error and precluding arbitrary official action. Given a budgetary constraint, the former approach would seek to maximize the number of guilty punished, while the latter would stress minimization of the number of innocent wrongly convicted. His dichotomy focuses attention on the cost in terms of incorrect convictions that society is willing to incur to achieve the punishment of various percentages of offenders.

10 P. COLLINS, DICKENS AND CRIME 74 (1968); J. TOBIAS, CRIME AND INDUSTRIAL SOCIETY IN THE 19TH CENTURY 207-08 (1967).
11 E. LIEBOW, TALLY'S CORNER; A STUDY OF NEGRO STREETCORNER MEN 43 n.8 (1967).
Assuming attention is restricted to instances where $y$ is small relative to $z$, the relationship that Packer stresses may be expressed within the framework of the model by writing $y$ as a function of $z$. The state can increase the disincentive impact of a given punishment by increasing $z$; if resources available for law enforcement do not vary, however, consequent gains will normally be at least partially offset by concomitant increases in $y$. Minimization of $y$ and maximization of $z$ are therefore inconsistent objectives. Over the relevant range not only $y$ but also the rate of change of $y$ should increase as $z$ increases, since high levels of $z$ normally imply punishment of the less clearly culpable. If error is a possibility in every adjudication, $y$ and $z$ can equal zero only simultaneously. A value of $z$ less than zero is most realistically interpreted as the result of misspecification of criminal conduct rather than as evidence of official perversity. An increase in $z$, like a decrease in $v$ or an increase in $w$, will raise $u$ by discouraging criminal behavior; the gain in welfare resulting from lessened criminality must nevertheless be balanced against the utility losses of those punished.

That these conclusions concerning the dependence of $y$ on $z$ do not hold for all mathematically possible values of the variables is evident from the fact that $z$ cannot exceed zero when $y$ is set equal to unity. The Bugandan solution to urban unrest probably approached this combination of maximal punishment and minimal discrimination as closely as any historical response:

Although the population of the capital must have run to tens of thousands, there is no sign that a city mob took any part in rebellions, as it may have done in some West African states. This is doubtless because very stern measures were taken to control the population of the capital. Periodically, and especially at times of political dissension, or when there were large numbers of rowdy men abroad in the streets, the royal executioners were sent out to ambush the streets of the capital and to seize and kill all persons who could not give a good explanation of their presence. Those arrested had not necessarily committed any crime: they are described as people who had no employment in the capital, and included innocent peasants who were bringing in food... It is therefore not surprising that the common people went in fear of their lives in the streets of the capital, and if they had to go there hastened home again as soon as their business was completed.\textsuperscript{13}

Such pathological extremes may properly be disregarded in analysis. Evidence of less extravagant British tastes is supplied by Hale, Blackstone and Paley, and Fortescue, who remonstrate unwillingness to

\textsuperscript{13} M. GLUCKMAN, POLITICS, LAW AND RITUAL IN TRIBAL SOCIETY 153 (1965).
sacrifice one innocent man to secure conviction of five, ten, and twenty offenders respectively. Bentham speaks of a ratio of one hundred to one. Paley nevertheless argues: "[C]ourts of justice should not be deterred . . . by every suspicion of danger, or by the mere possibility of confounding the innocent with the guilty.—They ought rather to reflect, that he who falls by a mistaken sentence, may be considered as falling for his country . . . ."14 His utilitarian pronouncement provides small comfort to those selected for patriotic sacrifice.15

Those uncertainties which prevent reduction of \( y \) to zero similarly preclude reliable estimation of its value. The value of \( z \), less difficult to compute, is surprisingly low for many categories of crime: in the United States in 1965 only twenty-two percent of reported serious offenses against property were resolved;16 even the regular user of marijuana risks but one chance in twenty of arrest in a given year.17 In any case, alteration of either \( y \) or \( z \) need not entail change in the other so long as total expenditures for law enforcement are not fixed.

IV

DETERRENCE

If the probability of criminal behavior were constant, changes in \( u, v, \) and \( w \) would affect welfare by altering the value of one or more of the possible outcomes; changes in \( y \) and \( z \) would affect welfare by altering the probabilities of these outcomes. Changes in the variables \( v, w, \) and \( z \), however, also affect \( x \), the probability of criminal behavior.


15 The competition in humanitarian superlatives has not escaped satirical treatment. Galbraith, reviewing reform following revolution in an imaginary Latin American state, records:

The Dean of the Faculty of Law of the University of Flores . . . took up the most advanced position. "The fundamental spirit of democracy," he told a meeting of the Faculty of Law a few days after the revolution, "proclaims that it is better that a hundred desperate criminals escape than that one innocent man be lodged in jail." He announced that he was inviting Professor Daniel Escobedo to spend a year in Puerto Santos making a study of the criminal code and mentioned their common Spanish antecedents. Somewhere he had got the impression that Escobedo was Dean of the Harvard Law School.


I have previously assumed $x$ a function of the disparity in attractiveness of the opportunities confronting the individual. This disparity, most conveniently expressed as the difference between the expected gains from illicit and licit conduct, may be obtained by subtraction of equation (1) from equation (2):

$$E_b - E_a = v - wz$$

Clearly $x$ will approach zero when $v - wz$ is a large negative number and unity when $v - wz$ is a large positive number; furthermore, $x$ should equal .5 when $v - wz$ is zero. The desired relationship may be written:

$$x = \frac{1}{1 + k^{v-wz}}$$

where the parameter $k$ must exceed unity. Although $x$ may be eliminated from equation (3) by substitution from equation (5), the complexity of the substituted term and its derivative makes this adjustment unrewarding.

The partial dependence of $x$ on the product $wz$ indicates that changes in the severity of sanctions and in the likelihood of their application to offenders have equivalent deterrent effects. The complementarity of these instruments of state action was noted by Bentham:

> Be the offence, be the punishment, what it may,—in proportion as you exclude this or that quibble, this or that device of technical procedure, by which a certain proportion of the whole number of delinquents are saved, and the probability of punishment in case of delinquency thereby diminished, you would put it in your power to make a correspondent and proportionable reduction in the magnitude of your punishment.

> What is the same thing in other words,—it is because your law is so full of quibbles, exclusionary rules, and other points of practice, by which impunity is given, and seen to be given, to known delinquents, that (the probability of punishment being subjected to constant diminution) delinquency receives proportionable increase: and, for combating it, the only other resource remaining, and the only resource that a quibble-loving lawyer will endure to hear of, is an increase of the magnitude of the punishment.18

Penologists have argued that "[c]ertainty of detection is far more important than severity of punishment."19 Their assertions of imbal-

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18 7 J. BENTHAM, WORKS 453 (J. Bowring ed. 1843).
19 SHAWCROSS, CRIME DOES PAY BECAUSE WE DO NOT BACK UP THE POLICE, N.Y. TIMES, June 13, 1965, § 6 (Magazine), at 44. SEE C. BECCARIA, ON CRIMES AND PUNISHMENTS 58 (H. Paolucci transl. 1968); W. PALEY, supra note 14, at 141.
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ance are in part the product of humanitarian assaults on severe sanctions associated with minor offenses in England until the middle of the nineteenth century. Recent sociological research affirming the deterrent effect of the criminal process but largely discounting the value of extended incarceration may be incorporated into the model by positing a nonlinear relationship between the prospective disutility of punishment to the individual and the period of his confinement: as this period increases the marginal impact of imprisonment may approach zero.  

Although a change in $x$ affects $E$ by altering the frequencies of actions with normally differentiated expected values, its more crucial impact occurs indirectly through consequent variation in $u$. The individual acts to maximize his own utility. While socialization may induce incorporation of the welfare of others as an important element in his preference function, some divergence between his interests and those of the community appears inevitable. Criminal law may be viewed as a means of internalizing externalities—by imposing penalties the state seeks to force the individual to bear the true costs of his conduct; as a result, behavior disadvantageous to others is discouraged. Reduced criminality may thus offer gain to all.

V

THE MACROECONOMIC CONSTRAINT

That $E$ is to be maximized does not, of course, imply that the variables on which it depends should be assigned extreme values. Conservative estimates place the direct cost of crime in the United States during 1965 at almost fifteen billion dollars; in the same period public and private expenditures for crime control exceeded six billion dollars. Even the least sophisticated among us recognizes that illicit activity can be reduced through commitment of additional resources. Demonstrating his intuitive grasp of the obvious, Mayor Daley himself has proclaimed: "If you have sufficient policemen, and people know that police are patrolling the streets, you will prevent much of your crime."  

In practice, outlays for crime control seem determined primarily by interaction between public reluctance to increase the level of tax-


21 PRESIDENT'S COMM'N ON LAW ENFORCEMENT & ADMINISTRATION OF JUSTICE, CRIME AND ITS IMPACT—AN ASSESSMENT 44 (1967).

22 QUOTATIONS FROM MAYOR DALEY 23 (P. Yessne ed. 1969).
ation and pressure for expansion generated from within the law enforcement industry. The police, abetted by their suppliers, are inclined to exaggerate the dangers of lawlessness in order to obtain increased appropriations and greater freedom of action. Simultaneously, of course, their symbiotic relationship with the criminal gives them a strong interest in his survival:

[O]ne state law-enforcement officer summed up the situation this way: "I've got a mortgage to pay off, so I've stopped fighting. Look, I'm a good guy. I'm for the good guys. I want to see the bad guys put in jail, like everyone else does. We don't want to rock the boat. There are too many guys in it. We'd all go down together. I guess you could say we're all part of the same establishment, the criminals and law enforcement."25

Maximization of social benefit entails reconciliation of outlays for law enforcement with competing allocational claims. Ideally, marginal investment in any aspect of any enterprise should yield a gain in utility precisely equal to the gain available through alternative investments and to the cost of the investment itself. If other investments are more or less profitable at the margin, diversion of funds to or from these opportunities is indicated; if the cost of incremental inputs exceeds or falls below the benefit derived, equalizing adjustment of total expenditure will prove advantageous. Rottenberg notes:

The gain from crimes prevented is the value of harm forestalled. If this could be done at zero cost, social welfare would be maximized by preventing all crime. Since crime prevention consumes resources that have alternative uses, however, the scale of the law enforcement industry should be pushed only to the margin where harm forestalled is equal in value to the resources employed in its achievement. The empirical magnitudes of costs and gains are likely to be such that some positive quantity of crime is optimal.26

In the model the opportunity cost of crime control is reflected in the level of u, a component of each of the four possible outcomes. Commitment of additional resources to the law enforcement sector implies

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their unavailability for other uses, and a consequent reduction in $u$ which may more than offset gains in $u$ resulting from lessened criminality. Higher levels of $z$ require further funding, detrimental changes in $y$, or more general circumscription of the personal liberties that are an important element of utility schedules. Increases in $w$ are also usually purchased: the cost of punishment is in part a function of the length of sentences, and associated safeguards may make execution more expensive than imprisonment for life.27

Substitution of monetary sanctions would reduce financial burdens but sacrifice equity to efficiency. Given a diverse population, punishment of all offenders by fines exactly balancing the external diseconomies of their behavior would yield an equilibrium paralleling that achieved under perfect competition. Since the sanction results in a transfer of utility rather than a loss of utility, penalized conduct would increase community welfare so long as profit to the offending individual exceeded his monetary sacrifice. The objection to such a system, also applicable to its economic analogue, is that under it benefits are allocated without regard to need. Burton noted an example of its inadequacies in British India:

He seems to have been the first to point out to Sir Charles—who was most reluctant to believe it—that though he had signed the death warrants of several rich convicted murderers, the actual man hanged was usually a poverty-stricken substitute hired in his stead. Burton interviewed one pauper "badal" who had agreed to be executed for a murder he had not committed and asked him why.

"Sain!" came the answer. "I have been a pauper all my life. My belly is empty. My wife and children are half starved. This is fate, but it is beyond my patience. I get two hundred and fifty rupees. With fifty I will buy rich food and fill myself before going out of the world. The rest I will leave to my family. What better can I do, Sain?"28

While condemnation of conduct as criminal usually supposes a determination by the community that its utility is negative, such judgments are hardly infallible: the Emperor Justinian, for example, forbade homosexual practices to prevent earthquakes.29 Packer, in particular, has asserted that the reach of the criminal law exceeds its grasp.30 Surely efforts to suppress the use of marijuana, arguably motivated by a puritan dread of unearned pleasure, deserve only low priority. The

more difficult question is whether resources diverted from control of marginally criminal activity should be reassigned within the law enforcement sector or applied to extrinsic goals.

The model itself supplies rules governing allocative efficiency with respect to crime control. The difficulty with allowing total outlays for law enforcement to be determined indirectly as a consequence of individual optimization of \( w, y, \) and \( z, \) the obvious instruments of state policy, is that expenditure patterns which fix the marginal utility of investment in other sectors are themselves distorted. Senator Russell, for instance, has supported appropriations for weaponry by appeal to perhaps the least convincing of objectives: “If we have to start over again with another Adam and Eve, then I want them to be Americans and not Russians—and I want them on this continent and not in Europe.”

Funds spent in Southeast Asia have yielded a negative return. Scrutiny of such seeming misspecifications of the values of the community is impossible within the framework of the model; the only recourse is to posit an improbable rationality.

**CONCLUSION**

Three functions summarize the relationships among the variables of the game of section I:

\[
\begin{align*}
(6) & \quad x = F_1 (v, w, z) \\
(7) & \quad u = F_2 (v, w, x, y, z) \\
(8) & \quad E = F_3 (u, v, w, x, y, z)
\end{align*}
\]

Maximization of \( E \) requires that the rates of change of \( E \) with respect to \( w, y, \) and \( z \) equal zero; if any is either positive or negative, further adjustment will yield gain. Thus, given appropriate second derivatives:

\[
\begin{align*}
(9) & \quad \frac{\partial E}{\partial w} = \frac{\partial F_3}{\partial w} + \frac{\partial F_3}{\partial x} \cdot \frac{\partial x}{\partial w} + \frac{\partial F_3}{\partial u} \cdot \frac{\partial u}{\partial w} = 0 \\
(10) & \quad \frac{\partial E}{\partial y} = \frac{\partial F_3}{\partial y} + \frac{\partial F_3}{\partial u} \cdot \frac{\partial u}{\partial y} = 0 \\
(11) & \quad \frac{\partial E}{\partial z} = \frac{\partial F_3}{\partial z} + \frac{\partial F_3}{\partial u} \cdot \frac{\partial u}{\partial z} + \frac{\partial F_3}{\partial u} \cdot \frac{\partial u}{\partial z} = 0
\end{align*}
\]

Here, $w$ and $z$ are shown to affect $E$ not only directly but in addition through their impact on $x$ and $u$, to which $E$ is also functionally related. Similarly, $E$ responds to alterations in $y$ both because of the direct relationship between these variables and as a consequence of the dependence of $u$ on $y$. Isolation of the effects of changes in $u$ caused directly and indirectly by changes in $w$ and $z$ is possible through expansion of terms from equations (9) and (11):

$$
\frac{\partial F_3}{\partial u} \cdot \frac{\partial u}{\partial w} = \frac{\partial F_3}{\partial F_2} \cdot \frac{\partial F_2}{\partial w} + \frac{\partial F_3}{\partial F_2} \cdot \frac{\partial F_2}{\partial x} \cdot \frac{\partial x}{\partial w}
$$

$$
\frac{\partial F_3}{\partial u} \cdot \frac{\partial u}{\partial z} = \frac{\partial F_3}{\partial F_2} \cdot \frac{\partial F_2}{\partial z} + \frac{\partial F_3}{\partial F_2} \cdot \frac{\partial F_2}{\partial x} \cdot \frac{\partial x}{\partial z}
$$

The response of $u$ is thus due in part to the impact of $w$ and $z$ on $x$.

Differentiation of equation (3) with respect to $u$, $w$, $x$, $y$, and $z$ permits substitution of more precise terms:

$$
\frac{\partial F_3}{\partial u} = 1
$$

$$
\frac{\partial F_3}{\partial w} = -y - xz
$$

$$
\frac{\partial F_3}{\partial x} = u - wz
$$

$$
\frac{\partial F_3}{\partial y} = -w
$$

$$
\frac{\partial F_3}{\partial z} = -wx
$$

The change in $x$ as $w$ and $z$ are altered may be obtained by differentiation of equation (5).

The probability of punishment regardless of culpability may be lowered without lessening the effectiveness of law enforcement efforts if additional resources are devoted to crime control. Thus, choice of an optimal level of the variable $y$ without regard to the relationship between $y$ and $z$ entails striking a balance between reduction of the number of nonoffenders mistakenly deprived of utility and increase in $u$ made possible by diversion of funds from efforts to discriminate between the guilty and the innocent. These effects are respectively summarized in the medial terms of equation (10):
The state may curtail illicit conduct by making the sanctions it imposes more severe or by augmenting the increase in the likelihood of punishment resulting from criminal behavior. As equations (9), (11), (12), and (13) indicate, benefit from such changes is reflected primarily if not exclusively in reduced injury from criminal activity. Higher levels of $w$ and $z$ indirectly affect $u$ and ultimately $E$ by lowering $x$:

$$\frac{\partial F_3}{\partial y} < 0$$

$$\frac{\partial F_3}{\partial u} \cdot \frac{\partial u}{\partial y} > 0$$

Concomitant loss is predominantly a consequence of two types of effects. First, higher levels of $w$ and $z$ entail greater disutility of punishment to those experiencing it and punishment of a greater proportion of offenders:

$$\frac{\partial F_3}{\partial F_2} \cdot \frac{\partial F_2}{\partial x} \cdot \frac{\partial x}{\partial w} > 0$$

$$\frac{\partial F_3}{\partial F_2} \cdot \frac{\partial F_2}{\partial x} \cdot \frac{\partial x}{\partial z} > 0$$

Second, the economic and noneconomic burdens of law enforcement seem likely to grow as $w$ and $z$ increase. Although the cost of punishment need not vary directly with its severity, it would appear in practice to do so:

$$\frac{\partial F_3}{\partial w} < 0$$

$$\frac{\partial F_3}{\partial z} < 0$$

Moreover, if changes in $z$ are achieved by reallocating resources while holding $y$ constant, one may write:

$$\frac{\partial F_3}{\partial F_2} \cdot \frac{\partial F_2}{\partial w} < 0$$

If $y$ is also permitted to vary, the result may be expressed by inclusion of an additional term in equation (11):
As shown in equation (16), the change in $F_3$ as $x$ changes is equal to $v - wz$. Since $v$ may or may not exceed $wz$, the direct impact in equations (9), (11), (12), and (13) of changes in the proportions of individuals selecting more or less profitable courses of action is indeterminate without additional assumptions:

\[
\frac{\partial F_3}{\partial x} \cdot \frac{\partial x}{\partial y} < 0
\]

Or:

\[
\frac{\partial F_3}{\partial x} \cdot \frac{\partial x}{\partial w} \geq 0
\]

Optimization requires both accurate estimation of community values and skillful manipulation of legal controls.